Load–Deformation Response of Cellular Silicone

Introduction

The engineering application of cellular rubbers requires that they support loads for long periods of time and follow differential deformations caused by temperature changes. It is essential that the load–temperature–time response of the cushion be known. It is desirable that the cushion response be predictable from the properties of the base rubber. This report describes a study of data that have been accumulated over several years from several sources. The purpose of the study was to define a relationship from these data that could be used to predict the load–deformation response of cellular silicone rubber.

Cellular silicone[1] is one of the most versatile of the cellular rubbers used in engineering cushions. It is made by incorporating a temporary filler into a base silicone rubber compound. This filler is subsequently removed to leave voids (cells) in the finished cushion. The technique permits control of the cell size, shape, and distribution. The load–deformation behavior of the rubber is obviously dependent on time and temperature. A controllable cell structure is desirable because a cellular structure complicates the load–deformation response of a cushion.

Discussion

The static load–deformation and creep behavior of many formulations of cellular silicone have been studied by a number of investigators. All the formulations are made from the same base silicone rubber; only the volume fraction of rubber is different. A typical load–deformation curve for a cellular silicone formulation is nonlinear. It can be described by two exponentials. Such curves have been measured for a number of formulations, and it seemed reasonable to expect that there was a relationship between them.

The load–deformation response of a cushion must be related to the properties of the rubber and its cellular configuration. Gunn[2] has proposed an empirical relationship (from the Gaussian statistical theory of rubber elasticity for full density rubber) between the applied load S and the resulting deformation:

$$S = A(d^{-2} - d) \tag{1}$$

where d is the relative height (h/h_0) of the deformed cushion. A is the apparent modulus of the cushion through the linear range. Gunn identifies the linear range with the compression of the cellular structure. Figure 1 shows some typical[3,4] load data for cellular silicone plotted against this relative height parameter. It is difficult to ascribe physical meaning to the parameter. The plots should be nearly linear to deformations of 50% with the densities of the cushions we studied. Since they do not appear linear, it is impractical to use the apparent modulus. Gunn relates the apparent modulus A of the cushion to the shear modulus G of the rubber and the cellular rubber density (ρ_2) by a power law:

$$A = G\rho_2^{\,n} \tag{2}$$

which he generalizes to

$$A = GV_r^{\,n} \tag{3}$$

where

$$V_r = \rho_2/\rho_1 \tag{4}$$

and ρ_1 is the rubber density. He found that a value of $n = 2$ fit the majority of his data and that the deviation increased for higher V_r.

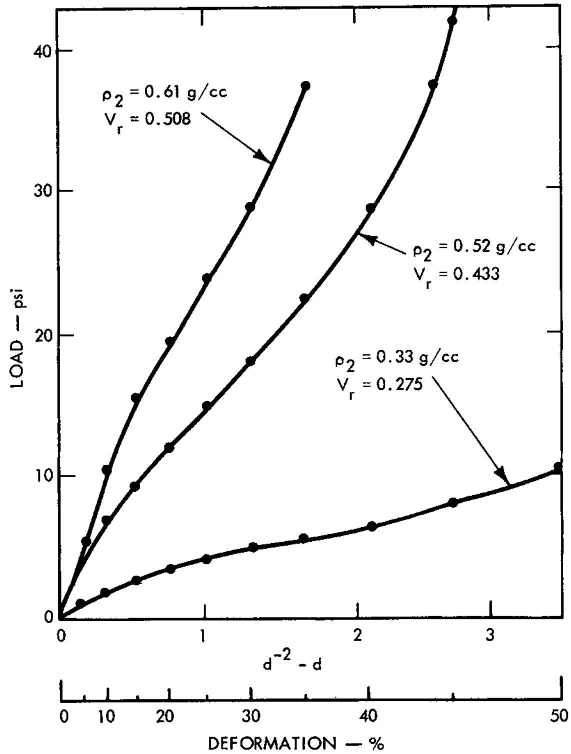

Fig. 1. Load plotted against $(d^{-2} - d)$ for three formulations of cellular silicone rubber but different volume fractions of rubber, V_r. The $V_r = \rho_2/\rho_1$, where ρ_2 is the density of the cellular silicone and ρ_1 (1.2 g./cc.) is the density of the rubber.

The deviations in Gunn's data for high V_r and the curvature shown in our data on Figure 1 caused us to suspect that the relationship was exponential. There is a linear log relationship between the applied load S and the deformation D up to about 40% deformation. Since 40% deformation is the practical limit of the current cellular silicone formulations, we confined our attention to this area. We observed a common slope to three sets of data on log coordinates. The curves are displaced on the ordinate proportionately to V_r. From these observations we propose an exponential relationship to relate the effective modulus, S/D (average of secants from the load–deformation curve taken in 5% steps from origin to 35% deformation), of a cellular silicone to the volume fraction of silicone rubber (V_r) contained in the cellular silicone.

$$\ln S = \ln D + f(V_r) \tag{5}$$

or

$$S/D = e^{f(V_r)} \tag{6}$$

The $f(V_r)$ can be expanded in a Taylor series. Using the three sets of data, we found that only the first term of the Taylor series is required in the range of deformations we are considering. Then we can write eq. (5) as

$$\ln S = \ln D + K(V_r) + C \tag{7}$$

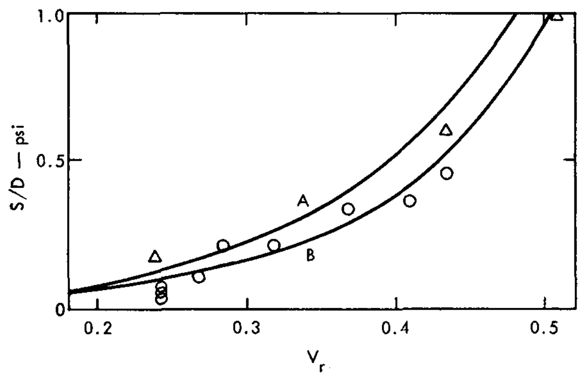

Fig. 2. Effective modulus, S/D, of the cellular silicone formulations plotted against volume fraction of rubber, V_r: (A) computed from $S/D = e^{8.2(V_r-0.479)}$ for the three sets of data, (B) computed from $S/D = 10^{-2}\,e^{9(V_r)}$ for all twelve sets of data.

or eq. (6) as

$$S/D = e^{K(V_r)-C} \tag{8}$$

The constants, K and C, were evaluated from the three sets of data. These three data are shown as open triangular points (△) on Figure 2 along with the computed curve.

The points shown as open circles (O) on Figure 2 were obtained from data of nine other cellular silicone formulations.[5] The function closely approximates all the data. The constants were evaluated by least squares for all twelve sets of data.

$$f(V_r) = 9(V_r) - 4.53 \tag{9}$$

or

$$S/D = 10^{-2}e^{9(V_r)} \tag{10}$$

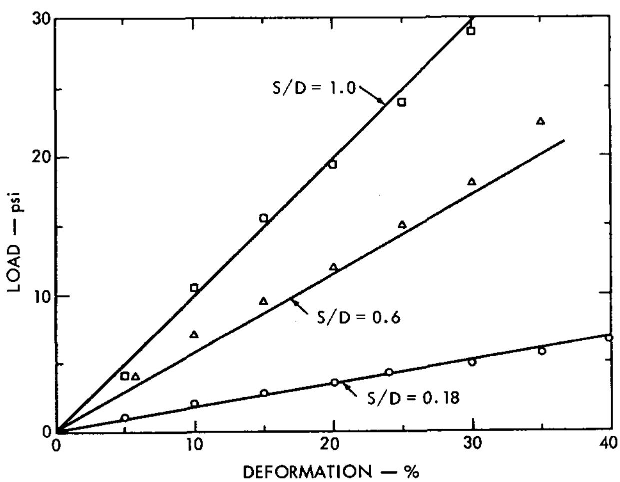

Fig. 3. Load–deformation curves for three formulations of cellular silicone rubber; the points are the experimentally measured values: (□) $V_r = 0.508$, (△) $V_r = 0.433$, (O) $V_r = 0.275$. The curves were computed from $S/D = 10^{-2}e^{9(V_r)}$.

Equation (10) is plotted through the experimentally determined points on Figure 2 for all twelve sets of data.

To test the adequacy of eq. (10) we have computed three S/D from three V_r. These are shown plotted with measured load–deformation points on Figure 3. We have used three independently measured sets of data to establish a relationship between the load-deformation behavior of a cellular silicone and its cell content. The relationship has been graduated with nine other sets of data and shown to describe all twelve. It appears that for a given rubber and a constant cell geometry, the load–deformation behavior of a cushion can be described by knowing the volume fraction of rubber of the cushion.

The load–deformation properties of a cellular silicone must also be related to the properties of the rubber. We would expect the constants in eq. (10) to shift with changes in cell geometry, cell size distribution, and open-to-closed cell ratio. We might also expect a shift as the properties of the base rubber were altered.

We would like to determine how responsive our relationship is to rubber properties. Kerner's equation[6] reduced for cellular materials predicts an effective shear modulus of 48 psi. Kerner's equation reduced for cellular material is

$$1/G_0 = 1/G_1\{1 + (V_2/V_1)[15(1 - \nu_1)/(7 - 5\nu_1)]\} \quad (11)$$

where G_0 is the effective modulus of the cellular material when the modulus of the rubber (G_1), the volume ratio of cell to rubber (V_2/V_1), and the Poisson's ratio of the rubber (ν_1) are known. The predicted shear modulus of 48 psi would imply a tensile modulus for the base rubber of 143 psi. Manufacturers values for the tensile modulus for the type of silicone rubber used in making cushions are from 250 to 350 psi. If we evaluate eq. (10) where $V_r = 1$, we would predict an effective modulus of 243 psi.

It has been observed that isochronous load–deformation curves taken from creep data after 1 hr. for cellular silicone closely resemble static load–deformation curves.[7] The creep behavior of cellular silicone can be adequately described by a power law in time.[8]

$$\epsilon = \epsilon_0 + mt^n \quad (12)$$

where ϵ is in units of deformation. The fitting technique used to determine ϵ_0 forces it to approximate the initial deformation.[9] Creep data of cellular silicone with $V_r = 0.508$ evaluated with eq. (12) at $t = 1$ hr. would predict a deformation of 27.4% from a 30-psi load. Taking full liberty with eq. (10) and writing

$$\epsilon_1 = S/10^2 e^{-9(V_r)} \quad (13)$$

we would compute a deformation of 31.1% after 1 hr. under 30 psi. The agreement is surprisingly good, since this deformation is approaching the limit where eq. (10) is applicable.

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